System-reservoir coupling derived by Maxwell's boundary conditions from weak to ultrastrong light-matter coupling regime

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(Dated: January 31, 2013)

In the standard theory of cavity quantum electrodynamics (QED), coupling between photons inside and outside a cavity (cavity system and photonic reservoir) is given conserving the total number of photons. However, when the cavity photons (ultra)strongly interact with atoms or excitations in matters, the system-reservoir coupling must be determined from a more fundamental viewpoint. Based on the Maxwell's boundary conditions in the QED theory for dielectric media, we derive the quantum Langevin equation and input-output relation, in which the total number of polaritons (not photons) inside the cavity and photons outside is conserved.

PACS numbers: 03.65.Yz,42.50.Pq,71.36.+c

The dissipation has long been discussed as an inevitable phenomenon in most systems, and the light is a typical target in the study of such open systems. When the light is confined in a cavity consisting of mirrors or distributed Bragg reflectors, discrete cavity modes are well identified, while they have finite broadenings due to the loss through the mirrors. In the standard theory of quantum optics [1, 2], coupling between the cavity modes and external photonic field is usually supposed as

$$\hat{H}_{\text{S-R}}^{\text{cavity}} = \sum_m \int \mathrm{d}\omega \ \mathrm{i}\hbar\sqrt{\frac{\kappa_m(\omega)}{2\pi}} \left[\hat{\alpha}^\dagger(\omega)\hat{a}_m - \hat{a}_m^\dagger\hat{\alpha}(\omega)\right]. \tag{1}$$

Here, $\hat{a}_m$ is the annihilation operator of a photon in m-th cavity mode, $\hat{\alpha}(\omega)$ is the one outside the cavity with frequency ω, and $\kappa_m(\omega)$ is the dissipation rate of the m-th mode. This expression has successfully reproduced experimental results, even when the cavity photons interact with atoms or excitations in matters. However, in the ultrastrong light-matter coupling regime, where the Rabi splitting frequency g is comparable to or larger than the transition frequency ω_{ex} of excitation in matter [3–14], we encounter a problem of the treatment of the system-reservoir coupling [15, 16]. This is because the rotating wave approximation (RWA) cannot be applied on the light-matter coupling, and the total number of photons and excitations is no longer conserved. Then, while the number of photons inside and outside the cavity is conserved in Eq. (1), we should reconsider the validity of this expression carefully.

In the ultrastrong light-matter coupling regime, even in the ground state of the coupled (polariton) system, there are virtual photons represented as a "squeezed" vacuum state [3]. As pointed out by Glauber and Lewenstein [17], such virtual photons exist even in a simple dielectric medium, and its "squeezing" character is different from that of the squeezed light in vacuum. Whereas the electromagnetic fields are certainly sub- and super-fluctuant in dielectrics, such a "squeezed" quantum fluctuation recovers to the one of the coherent or vacuum state when the fields escape from the dielectrics to the vacuum. In this way, even in the ultrastrong light-matter coupling regime, the polaritons simply represent the electromagnetic fields in dielectrics, and we cannot generate nonclassical light outside the cavity at least in the linear optical process with classical inputs. Certainly, classical outputs are obtained by classical inputs at least in the approach of Langevin equations [4, 17]. However, when we simply suppose Eq. (1), we encounter a delicate problem: Even if the outside is in the vacuum, since the virtual photons inside the cavity can escape to the outside, the polariton system is inevitably excited [16].

Instead of supposing Eq. (1), the excitation of polaritons by the vacuum can be avoided by considering

$$\hat{H}_{\text{S-R}}^{\text{MBC}} = \sum_j \int \mathrm{d}\omega \ \mathrm{i}\hbar\sqrt{\frac{\kappa_j(\omega)}{2\pi}} \left[\hat{\alpha}^\dagger(\omega)\hat{p}_j - \hat{p}_j^\dagger\hat{\alpha}(\omega)\right], \tag{2}$$

where $\hat{p}_j$ annihilates a polariton in j-th eigen-mode and $\kappa_j(\omega)$ is its dissipation rate. Although this expression can be obtained by applying the RWA on Eq. (1) in the polariton base [15], we will show that expression (2) is rather natural based on the quantum electrodynamics (QED) theory for dielectric media [18–24], then the polariton system is in principle not excited by the vacuum even in the absence of the RWA. We will also compare the dissipation rate derived from Eq. (1) with the RWA and that from Eq. (2), which show qualitatively different behaviors with the increase of light-matter coupling into the ultrastrong regime.

First, we consider a loss-less homogeneous dielectric medium, in which photons interact with infinite-mass excitations [3, 25]. The Hamiltonian is represented as

$$\begin{aligned}\hat{H}_{\text{p}} = \sum_{k=-\infty}^{\infty} \Big\{ & \hbar c|k|\hat{a}_k^\dagger\hat{a}_k + \hbar\omega_{\text{ex}}\hat{b}_k^\dagger\hat{b}_k \\ & + \mathrm{i}\hbar g_k(\hat{a}_k + \hat{a}_{-k}^\dagger)(\hat{b}_{-k} - \hat{b}_k^\dagger) \\ & + \hbar D_k(\hat{a}_k + \hat{a}_{-k}^\dagger)(\hat{a}_{-k} + \hat{a}_k^\dagger) \Big\}.\end{aligned} \tag{3}$$

Here, $\hat{a}_k$ and $\hat{b}_k$ are annihilation operators of a photon and an excitation with wavenumber k in z direction, respectively, and satisfy $[\hat{a}_k, \hat{a}_{k'}^\dagger] = [\hat{b}_k, \hat{b}_{k'}^\dagger] = \delta_{k,k'}$. Using the photon operator, the vector potential is represented as

$$\hat{A}(z) = \sum_{k=-\infty}^{\infty} \sqrt{\frac{\hbar}{2\varepsilon_0 c|k|SL}} \left(\hat{a}_k + \hat{a}_{-k}^\dagger\right) \mathrm{e}^{\mathrm{i}kz}, \tag{4}$$

where c is the speed of light in vacuum, ε_0 is the vacuum permittivity, S is the area in $x-y$ plane, and L is the length in z direction. ω_{ex} is the frequency of excitations, g_k is the light-matter coupling strength, and the coefficient of the last term is $D_k = {g_k}^2/\omega_{\mathrm{ex}}$. Introducing the annihilation operators of lower and upper ($j = L$ and U) polaritons as

$$\hat{p}_{j,k} = w_{jk}\hat{a}_k + x_{jk}\hat{b}_k + y_{jk}\hat{a}_{-k}^\dagger + z_{jk}\hat{b}_{-k}^\dagger, \tag{5}$$

we can diagonalize Eq. (3) [3, 25]:

$$\hat{H}_{\mathrm{p}} = \sum_{j=L,U} \sum_{k=-\infty}^{\infty} \hbar\omega_{j,k}\hat{p}_{j,k}^\dagger \hat{p}_{j,k}. \tag{6}$$

The eigen-frequencies ω_{jk} and coefficients $\{w_{jk}, x_{jk}, y_{jk}, z_{jk}\}$ are determined by $[\hat{p}_{jk}, \hat{H}_{\mathrm{p}}] = \hbar\omega_{jk}\hat{p}_{jk}$ and $[\hat{p}_{jk}, \hat{p}_{j'k'}^\dagger] = \delta_{jj'}\delta_{k,k'}$, and $\omega = \omega_{jk}$ satisfies

$$\frac{c^2k^2}{\omega^2} = \varepsilon_{\mathrm{p}}(\omega) = 1 + \frac{4\pi\beta{\omega_{\mathrm{ex}}}^2}{\omega_{\mathrm{ex}} - (\omega + \mathrm{i}0^+)^2}. \tag{7}$$

Here, $\varepsilon_{\mathrm{p}}(\omega)$ is the dielectric function of the polariton medium [21, 25], and we suppose the coefficient $4\pi\beta = 4c|k|{g_k}^2/{\omega_{\mathrm{ex}}}^3$ does not depend on k for simplicity. By using the polariton operator, the positive-frequency component of vector potential (4) is rewritten as [26]

$$\hat{A}^+(z) = \sum_{j=L,U} \sum_{k=-\infty}^{\infty} \sqrt{\frac{\hbar v_{\mathrm{g}}(\omega_{jk})}{2\varepsilon_0 c\omega_{jk} n_{\mathrm{p}}(\omega_{jk})SL}} \hat{p}_{jk}\mathrm{e}^{\mathrm{i}kz}. \tag{8}$$

Here, $n_{\mathrm{p}}(\omega) = \sqrt{\varepsilon_{\mathrm{p}}(\omega)}$ is the refractive index, and

$$v_{\mathrm{g}}(\omega) = \frac{\partial\omega}{\partial k} = \frac{c}{n_{\mathrm{p}}(\omega) + \omega(\partial n_{\mathrm{p}}/\partial\omega)} \tag{9}$$

is the group velocity. In this way, polaritons represent the eigen-states in dielectrics even in the ultrastrong light-matter coupling regime $4\pi\beta \gtrsim 1$. As discussed in Ref. [17], the quantum fluctuation of the electromagnetic fields is modulated in dielectrics, which can also be obtained from Eq. (8) [26].

Next, in order to introduce boundaries of the polariton system, we employ the QED theory for inhomogeneous media [20–24]. We simply consider an one-dimensional (1D) system with dielectric function $\varepsilon(z,\omega)$ depending on position z and frequency ω satisfying the Kramers-Kronig relation, and the electric and magnetic fields are in the $x-y$ plane. The positive-frequency component $\hat{A}^+(z,\omega)$ of vector potential in this system obeys

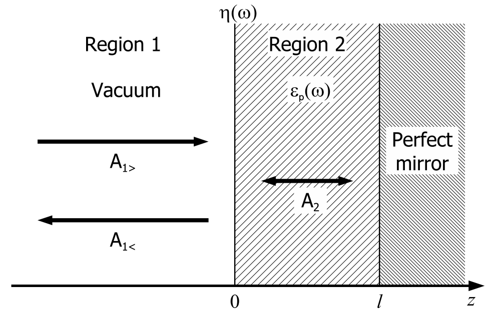

FIG. 1. Sketch of the considered one-dimensional system with dielectric function in Eq. (13).

$$\left[\frac{\partial^2}{\partial z^2} + \frac{\omega^2}{c^2}\varepsilon(z,\omega)\right]\hat{A}^+(z,\omega) = -\mu_0\hat{J}_{\mathrm{N}}(z,\omega). \tag{10}$$

This has exactly the same form as the wave equation derived from the Maxwell equations. Here, μ_0 is the vacuum permeability, and the quantum fluctuation of the electromagnetic fields is described by the noise current operator $\hat{J}_{\mathrm{N}}(z,\omega)$ satisfying

$$[\hat{J}_{\mathrm{N}}(z,\omega), \hat{J}_{\mathrm{N}}(z',\omega')^\dagger] = \delta(\omega-\omega')\delta(z-z')\frac{\varepsilon_0\hbar\omega^2}{\pi S}\mathrm{Im}[\varepsilon(z,\omega)]. \tag{11}$$

The electric and magnetic fields are represented as $\hat{E}(z,t) = -(\partial/\partial t)\hat{A}(z,t)$ and $\hat{B}(z,t) = (\partial/\partial z)\hat{A}(z,t)$, respectively, and their positive-frequency components satisfy the Maxwell equations

$$\frac{\partial}{\partial z}\hat{E}^+(z,\omega) = \mathrm{i}\omega\hat{B}^+(z,\omega), \tag{12a}$$

$$-\frac{1}{\mu_0}\frac{\partial}{\partial z}\hat{B}^+(z,\omega) = -\mathrm{i}\omega\hat{D}^+(z,\omega). \tag{12b}$$

The latter equation is equivalent with Eq. (10), and the displacement field includes the noise current as $\hat{D}^+(z,\omega) = \varepsilon_0\varepsilon(z,\omega)\hat{E}^+(z,\omega) + (\mathrm{i}/\omega)\hat{J}_{\mathrm{N}}(z,\omega)$. Based on this formalism [20–24], the positive- and negative-frequency components never mix with each other at least in the linear optical process. Then, for polariton system confined in an optical cavity, the annihilation operator $\hat{p}_j$ of polariton couples with $\hat{\alpha}(\omega)$ of photon outside the cavity, and they never couple with creation operators [$\hat{p}_j^\dagger$ and $\hat{\alpha}^\dagger(\omega)$] in the linear optical process. Next, we explicitly consider a cavity system, and derive the quantum Langevin equation and input-output relation from the above Maxwell equations.

As discussed in Ref. [27], we consider a cavity system shown in Fig. 1, where the dielectric function is given as

$$\varepsilon(z,\omega) = \eta(\omega)\delta(z) + \begin{cases} 1 & z < 0 \\ \varepsilon_{\mathrm{p}}(\omega) & 0 < z < l \end{cases} \tag{13}$$

There is a perfect mirror at $z = l$, and the other mirror is placed at $z = 0$. $\eta(\omega)$ determines the transparency between the cavity and the outside. For the solution to Eq. (10), we suppose the vector potential in the two regions as depicted in Fig. 1:

$$\hat{A}_1^+(z,\omega) = \hat{A}_{1>}^+(\omega)\mathrm{e}^{\mathrm{i}(\omega/c)z} + \hat{A}_{1<}^+(\omega)\mathrm{e}^{-\mathrm{i}(\omega/c)z}, \tag{14a}$$

$$\hat{A}_2^+(z,\omega) = \hat{A}_2^+(\omega)\sin[k_\mathrm{p}(\omega)(l-z)], \tag{14b}$$

where $k_\mathrm{p}(\omega) = n_\mathrm{p}(\omega)\omega/c$. Since the electric field is completely zero at the boundary with the perfect mirror, the intra-cavity mode has no amplitude at $z = l$. Here, as discussed in Ref. [20–24], the incoming field $\hat{A}_{1>}$ can be simply derived as

$$\hat{A}_{1>}^+(\omega) = \sqrt{\frac{\hbar}{4\pi\varepsilon_0 c\omega S}}\hat{a}_>(\omega), \tag{15}$$

where $\hat{a}_>(\omega)$ is defined as

$$\hat{a}_>(\omega) = \mathrm{i}\sqrt{\frac{\pi\mu_0 cS}{\hbar\omega}}\int_{-\infty}^{0}\mathrm{d}z\ \mathrm{e}^{-\mathrm{i}(\omega/c)z}\hat{J}_\mathrm{N}(z,\omega). \tag{16}$$

From Eq. (11), this operator satisfies $[\hat{a}_>(\omega),\hat{a}_>^\dagger(\omega')] = \delta(\omega-\omega')$, and it corresponds to the annihilation operator of an incoming photon.

Next, we consider boundary conditions determining $\hat{A}_{1<}^+(\omega)$ and $\hat{A}_2^+(\omega)$. From the continuous condition at $z=0$ derived from Eq. (12a), we get

$$\hat{A}_{1>}^+(\omega) + \hat{A}_{1<}^+(\omega) = \hat{A}_2^+(\omega)\sin[k_\mathrm{p}(\omega)l]. \tag{17a}$$

From the integral form of Eq. (12b), we also get

$$\begin{aligned}&[\hat{A}_{1>}^+(\omega) - \hat{A}_{1<}^+(\omega)] - \mathrm{i}n_\mathrm{p}(\omega)\hat{A}_2^+(\omega)\cos[k_\mathrm{p}(\omega)l]\\ &\quad = -\mathrm{i}\Lambda(\omega)\hat{A}_2^+(\omega)\sin[k_\mathrm{p}(\omega)l],\end{aligned} \tag{17b}$$

where $\Lambda(\omega) = \eta(\omega)\omega/c$. Solving these Maxwell's boundary conditions (MBCs), we get

$$\hat{A}_2^+(\omega) = \frac{2\hat{A}_{1>}^+(\omega)}{[1-\mathrm{i}\Lambda(\omega)]\sin[k_\mathrm{p}(\omega)l] + \mathrm{i}n_\mathrm{p}(\omega)\cos[k_\mathrm{p}(\omega)l]}, \tag{18}$$

and $\hat{A}_{1<}^+(\omega)$ is expressed by Eq. (17a). They are also derived from Eq. (10) by using the Green's function [26]. As seen in Eq. (18), we can find that resonances are obtained at $\omega = \Omega_m$ satisfying

$$\tan[n_\mathrm{p}(\Omega_m)\Omega_m l/c] = n_\mathrm{p}(\Omega_m)/\Lambda(\Omega_m) \tag{19}$$

and the frequency broadening is proportional to $\Lambda(\Omega_m)^{-2}$. Here, as discussed in Ref. [27], we consider the good cavity limit $\Lambda(\Omega_m) \gg n_\mathrm{p}(\Omega_m)$, and we suppose that $\Lambda(\omega)$ varies only slightly around $\omega = \Omega_m$ for simplicity. Under these assumptions, around the resonance $\omega \sim \Omega_m$, Eqs. (18) and (17a) are approximately rewritten as

$$A_2^+(\omega) = \sqrt{\frac{2v_\mathrm{g}(\Omega_m)}{n_\mathrm{p}(\Omega_m)l}}\frac{\mathrm{i}\sqrt{\kappa_\mathrm{MBC}(\Omega_m)}}{\omega-\Omega_m+\mathrm{i}\kappa_\mathrm{MBC}(\Omega_m)/2}A_{1>}^+(\omega), \tag{20a}$$

$$A_{1>}^+(\omega) + A_{1<}^+(\omega) = \sqrt{\frac{n_\mathrm{p}(\Omega_m)l}{2v_\mathrm{g}(\Omega_m)}}\sqrt{\kappa_\mathrm{MBC}(\Omega_m)}A_2^+(\omega), \tag{20b}$$

where the dissipation rate $\kappa_\mathrm{MBC}(\omega)$ is defined as

$$\kappa_\mathrm{MBC}(\omega) = \frac{2n_\mathrm{p}(\omega)v_\mathrm{g}(\omega)}{\Lambda(\omega)^2 l}. \tag{21}$$

In the semi-infinite region 1, $\hat{A}_{1<}^+(\omega)$ is expressed by the annihilation operator $\hat{a}_{1<}(\omega)$ of an outgoing photon as

$$\hat{A}_{1<}^+(\omega) = \sqrt{\frac{\hbar}{4\pi\varepsilon_0 c\omega S}}\hat{a}_{1<}(\omega). \tag{22}$$

On the other hand, in the finite region 2, in the similar way for deriving Eq. (8), the expression of $\hat{A}_2^+(z)$ is represented in the good cavity limit:

$$\hat{A}_2^+(z) = \sum_{j=L,U}\sum_{m=1}^{\infty}\sqrt{\frac{\hbar v_\mathrm{g}(\Omega_{jm})}{\varepsilon_0 c\Omega_{jm}n_\mathrm{p}(\Omega_{jm})Sl}}\hat{p}_{jm}\sin[k_m(l-z)], \tag{23}$$

where $k_m = m\pi/l$, $\Omega_{jm} = \omega_{j,k=k_m}$ and $[\hat{p}_{jm},\hat{p}_{j'm'}^\dagger] = \delta_{j,j'}\delta_{m,m'}$. Then, from Eqs. (15), (22), and (23), Eqs. (20) are rewritten in the good cavity limit as

$$\hat{p}_{jm}(\omega) = \frac{\mathrm{i}\sqrt{\kappa_\mathrm{MBC}(\Omega_{jm})}}{\omega-\Omega_{jm}+\mathrm{i}\kappa_\mathrm{MBC}(\Omega_{jm})/2}\hat{a}_\mathrm{in}(\omega), \tag{24a}$$

$$\hat{a}_\mathrm{in}(\omega) + \hat{a}_\mathrm{out}(\omega) = \sum_{j=L,U}\sum_{m=1}^{\infty}\sqrt{\kappa_\mathrm{MBC}(\Omega_{jm})}\hat{p}_{jm}(\omega). \tag{24b}$$

where the input and output operators are defined as $\hat{a}_\mathrm{in}(\omega) = \hat{a}_{1>}(\omega)/\sqrt{2\pi}$ and $\hat{a}_\mathrm{out}(\omega) = \hat{a}_{1<}(\omega)/\sqrt{2\pi}$. Eqs. (24) have certainly the same form as the quantum Langevin equation and input-output relation derived from Eq. (2) by the well-known treatment in quantum optics [1, 2]. In this way, we should suppose Eq. (2) for the coupling between the polariton system and the photonic reservoir, instead of Eq. (1) with the RWA.

To check the consistency with the well-known discussions [1, 2], we suppose the simplified case where the resonance frequency of the lowest bare cavity mode is tuned to ω_ex, and we calculate the dissipation rates derived from Eq. (21). For simplicity, we suppose $\Lambda(\omega)$ is constant in the frequency range of interest. In the good cavity limit $\Lambda \gg 1$, the cavity length is determined as $l_0 \simeq \pi c/\omega_\mathrm{ex}$ satisfying $\tan(\omega_\mathrm{ex}l/c) \ll 1$. Then, from Eq. (21), the dissipation rate of bare cavity mode is obtained as $\kappa_0 = 2c/\Lambda^2 l_0$. From Eq. (19), the frequencies of lowest upper and lower polariton modes ($m = 1$ is omitted in following discussion) are determined by $n(\Omega_{L,U}) = \omega_\mathrm{ex}/\Omega_{L,U}$, and Eq. (21) is rewritten as

$$\kappa_\mathrm{MBC}(\Omega_{L,U}) = \frac{\kappa_0}{1+(\Omega_{L,U}/\omega_\mathrm{ex})^2}. \tag{25}$$

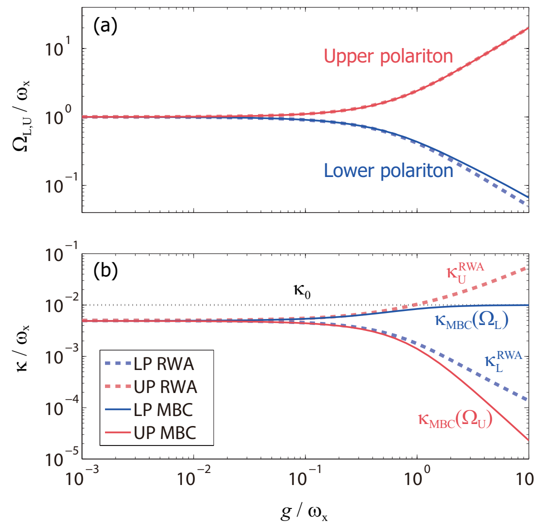

FIG. 2. (a) Frequencies of lower and upper polaritons (LP and UP) are plotted versus $g/\omega_{\rm ex}$. Dashed curves are calculated by Hamiltonian (3), while solid curves are obtained by Eq. (19). (b) Dissipation rates of the polaritons are plotted. Dashed curves are calculated from Eq. (1) with the RWA and $\kappa_0/\omega_{\rm ex} = 10^{-2}$. Solid curves are obtained from Eq. (21) (derived by the MBCs) by supposing $\varLambda = 7.822$ and l tuned for satisfying $\tan(\omega_{\rm ex} l/c) = 1/\varLambda$. Dotted line represents $\kappa_0/\omega_{\rm ex}$.

If the light-matter coupling is not so strong $4\pi\beta \ll 1$, we get $\varOmega_{L/U} \simeq \omega_{\rm ex} \pm g$, and $\kappa_{\rm MBC}(\varOmega_{L,U})$ is approximately a half of κ_0. Then, in the weak and normally strong coupling regimes, the resonance frequencies of cavity polaritons and their dissipation rates certainly agree with the well-known ones.

In Fig. 2, we plot (a) frequencies $\varOmega_{L,U}$ of lower and upper polaritons (LP and UP) and (b) their dissipation rates as functions of $g_{k=\omega_{\rm ex}/c}/\omega_{\rm ex} = \sqrt{4\pi\beta}/2$. For dashed curves, $\varOmega_{L,U}$ are calculated by diagonalizing Eq. (3) at $k = \omega_{\rm ex}/c$, and the dissipation rates are derived from Eq. (1) with $\kappa_{m=1}(\omega) = \kappa_0 = 10^{-2}\omega_{\rm ex}$ with the RWA on the system-reservoir coupling. For solid curves, $\varOmega_{L,U}$ and $\kappa_{\rm MBC}(\varOmega_{L,U})$ are determined by Eqs. (19) and (21), respectively, for $\varLambda = 7.822$ corresponding to $\kappa_0/\omega_{\rm ex} = 10^{-2}$. As seen in Fig. 2, the two approaches give almost the same results in the weak and normally strong light-matter coupling regimes $g \ll \omega_{\rm ex}$, where the frequencies are given as $\varOmega_{L,U} \simeq \omega_{\rm ex} \pm g$ and the dissipation rates are $\kappa_0/2$. In the ultrastrong light-matter coupling regime $g \gtrsim \omega_{\rm ex}$, concerning the resonance frequencies $\varOmega_{L,U}$, the two approaches agree well with each other at least qualitatively, while there is a discrepancy for the lower polariton frequency $\varOmega_L$ (this is because $\varOmega_L$ closes to the dissipation rate κ_0). However, as seen in Fig. 2(b), the dissipation rates obtained by the two approaches show qualitatively different behaviors. The solid curves can be well fitted by Eq. (25) [28], while the dashed curves are expressed as $\kappa_{L,U}^{\rm RWA} = |w_{L,U}|^2\kappa_0$ [15, 16], where coefficient $w_{L,U}$ appearing in Eq. (5) is determined by the diagonalization [29]. In this way, in the ultrastrong light-matter coupling regime, Eq. (1) give incorrect dissipation rates compared with Eq. (21) determined by the MBCs.

In summary, based on the QED theory for dielectric media [18–24], from the weak to ultrastrong light-matter coupling regime, the quantum Langevin equation (24a) of discrete cavity polariton modes and the input-output relation (24b) are derived in the good cavity limit. They suggest that Eq. (2) is appropriate for the coupling between the polariton system and the photonic reservoir in the good cavity case [30]. This result is also consistent with Ref. [19, 24], the discussion of the universe modes [31–34], and the Freshbach's projector approach [35, 36], while empty cavities were mainly discussed. Although the counter-rotating terms such as $\alpha(\omega)\hat{p}_j$ and $\hat{p}_j^\dagger\hat{\alpha}^\dagger(\omega)$ might appear in general due to the overlap between the cavity modes [32, 35], they are negligible thanks to the quality of the cavity, not to the RWA on system-reservoir coupling. As seen in Fig. 2(b), the dissipation rates of the cavity polaritons derived from Eq. (1) with the RWA show the qualitatively different behavior compared to Eq. (21) derived by the QED theory (MBCs), whereas they agree well with each other in the weak and normally strong light-matter coupling regimes. This can be understood in the sense of perturbation. If the light-matter coupling is stronger than the coupling with environment, we should diagonalize the cavity system first, then the coupling with environment is treated perturbatively. Then, Eq. (2) is appropriate in general for the entire coupling regimes [37]. If we want to discuss also the bad cavity case, we have to explicitly consider the MBCs (17), and the simplified expressions (1) and (2) cannot be used. Whereas we simply supposed the cavity system depicted in Fig. 1, specific cavity structures can also be considered based on the QED theory for dielectrics [20, 22–24]. If we consider a loss or a gain inside the cavity by supposing the imaginary part of the dielectric function $\varepsilon_{\rm p}(\omega)$, the Langevin equation (24a) would have another noise term expressed by $\hat{J}_{\rm N}(0 < z < l, \omega)$. This work remains for the future. Further, whereas the simple dielectric (polariton) medium was discussed in this letter, for cavity systems with an atom (nonlinear systems) and for the Dicke model (the gauge invariance is broken) [38–40], it is still open to dispute whether the system-reservoir coupling is generally expressed by the eigen-states of the cavity system as in Eq. (2). However, the result in this letter suggests a universal policy: The system-reservoir coupling must be determined from a fundamental viewpoint after diagonalizing strongly coupled composite systems. Even for the superconducting circuits [11–13], the system-reservoir coupling under the ultrastrong coupling between artificial atoms and resonator modes should be determined by an appropriate microscopic description [41].

M. B. thanks to Howard Carmichael and Pierre Nataf

for informing important articles. This work was supported by KAKENHI (No. 20104008 and No. 24-632) and the JSPS through its FIRST Program.

Quantum fluctuation in homogeneous dielectric system

In an homogeneous medium with dielectric function $\varepsilon(z,\omega)=\varepsilon(\omega)$, from the wave equation (10), the vector potential is expressed as

$$\hat{A}^{+}(z,\omega)=\hat{A}^{+}_{>}(z,\omega)+\hat{A}^{+}_{<}(z,\omega), \tag{26}$$

where $\hat{A}^{+}_{>}$ and $\hat{A}^{+}_{<}$ are forward and backward fields defined as

$$\hat{A}^{+}_{>}(z,\omega)=-\mu_0\int_{-\infty}^{z}\mathrm{d}z'\ \frac{\mathrm{e}^{\mathrm{i}k(\omega)(z-z')}}{\mathrm{i}2k(\omega)}\hat{J}_{\mathrm{N}}(z',\omega), \tag{27a}$$

$$\hat{A}^{+}_{<}(z,\omega)=-\mu_0\int_{z}^{\infty}\mathrm{d}z'\ \frac{\mathrm{e}^{-\mathrm{i}k(\omega)(z-z')}}{\mathrm{i}2k(\omega)}\hat{J}_{\mathrm{N}}(z',\omega). \tag{27b}$$

Since the noise current density $\hat{J}_{\mathrm{N}}(z,\omega)$ has the local correlation (commutable for different positions $z\neq z'$), the forward field $\hat{A}_{>}(z)$ and backward one $\hat{A}_{<}(z')$ are commutable for $z<z'$

$$[\hat{A}^{+}_{>}(z,\omega),\hat{A}^{-}_{<}(z',\omega')]=0\quad\text{for } z<z'. \tag{28}$$

The quantum fluctuation of the vector potential in the frequency domain is obtained as

$$[\hat{A}^{+}_{>}(z,\omega),\hat{A}^{-}_{>}(z',\omega')]=\delta(\omega-\omega')\frac{\hbar}{4\pi\varepsilon_0 c\omega S}\frac{\mathrm{Re}[n(\omega)]}{|n(\omega)|^2}\mathrm{e}^{\mathrm{i}\mathrm{Re}[k(\omega)](z-z')-\mathrm{Im}[k(\omega)]|z-z'|}. \tag{29}$$

When we define the spatial Fourier transform for wavenumber $q>0$ as

$$\hat{A}^{+}_{q}=\frac{1}{\sqrt{L}}\int\mathrm{d}z\ \mathrm{e}^{-\mathrm{i}qz}\hat{A}^{+}_{>}(z,\omega), \tag{30}$$

the quantum fluctuation of this mode in the frequency domain is obtained as

$$[\hat{A}^{+}_{q}(\omega),\hat{A}^{-}_{q'}(\omega')]=\delta(\omega-\omega')\delta_{q,q'}\frac{2\mathrm{Im}[k(\omega)]}{|q-k(\omega)|^2}\frac{\hbar}{4\pi\varepsilon_0 c\omega S}\frac{\mathrm{Re}[n(\omega)]}{|n(\omega)|^2} \tag{31a}$$

$$=\delta(\omega-\omega')\delta_{q,q'}\frac{\hbar}{4\pi\varepsilon_0 c\omega S}\frac{\mathrm{Re}[n(\omega)]}{|n(\omega)|^2}\left[\int_{-\infty}^{0}\mathrm{d}z\ \mathrm{e}^{\mathrm{i}[q-k(\omega)]z}+\int_{0}^{\infty}\mathrm{d}z\ \mathrm{e}^{\mathrm{i}[q-k(\omega)^*]z}\right]. \tag{31b}$$

In the loss-less limit ($\mathrm{Im}[\varepsilon(\omega)]\to 0$), we get

$$[\hat{A}^{+}_{q}(\omega),\hat{A}^{-}_{q'}(\omega')]=\delta(\omega-\omega')\delta_{q,q'}\delta(q-k(\omega))\frac{\hbar}{2\varepsilon_0 c\omega S}\frac{1}{n(\omega)}. \tag{32}$$

Then, the equal-time quantum fluctuation of this mode is finally written as

$$[\hat{A}^{+}_{q},\hat{A}^{-}_{q'}]=\delta_{q,q'}\int_{0}^{\infty}\mathrm{d}\omega\ \delta(q-k(\omega))\frac{\hbar}{2\varepsilon_0 c\omega S}\frac{1}{n(\omega)}$$

$$=\delta_{q,q'}\frac{\hbar}{2\varepsilon_0 cqS}\frac{1}{n(\varOmega_q)}, \tag{33a}$$

where $\varOmega_q=cq/n(\varOmega_q)$. Therefore, in the loss-less dielectric media, the quantum fluctuation of the vector potential is modified by the factor of $n(\varOmega_q)^{-1}$. The fluctuations of the other electromagnetic fields are obtained as

$$[\hat{E}^{+}_{q},\hat{E}^{-}_{q'}]=\delta_{q,q'}\frac{\hbar cq}{2\varepsilon_0 S}\frac{1}{n(\varOmega_q)^3}, \tag{33b}$$

$$[\hat{B}^{+}_{q},\hat{B}^{-}_{q'}]=\delta_{q,q'}\frac{\hbar q}{2\varepsilon_0 cS}\frac{1}{n(\varOmega_q)}, \tag{33c}$$

$$[\hat{D}^{+}_{q},\hat{D}^{-}_{q'}]=\delta_{q,q'}\frac{\hbar cq}{2\varepsilon_0 S}n(\varOmega_q). \tag{33d}$$

The dependence of these fields on $n(\Omega_q)$ is exactly the same as the one discussed in Ref. [17]. Then, the electromagnetic fields are sub-fluctuant or super-fluctuant in dielectrics compared to the case in vacuum.

The forward and backward fields (27) are rewritten as

$$\hat{A}^{+}_{>}(z,\omega) = \sqrt{\frac{\hbar}{4\pi\varepsilon_0 c\omega \mathrm{Re}[n(\omega)]S}} \frac{\mathrm{Re}[n(\omega)]}{n(\omega)} \mathrm{e}^{\mathrm{i}\mathrm{Re}[k(\omega)]z} \hat{a}_{>}(z,\omega), \tag{34a}$$

$$\hat{A}^{+}_{<}(z,\omega) = \sqrt{\frac{\hbar}{4\pi\varepsilon_0 c\omega \mathrm{Re}[n(\omega)]S}} \frac{\mathrm{Re}[n(\omega)]}{n(\omega)} \mathrm{e}^{-\mathrm{i}\mathrm{Re}[k(\omega)]z} \hat{a}_{<}(z,\omega), \tag{34b}$$

where operators $\hat{a}_{>}$ and $\hat{a}_{<}$ are defined as

$$\hat{a}_{>}(z,\omega) = \mathrm{i}\sqrt{\frac{\pi\mu_0 cS}{\hbar\omega \mathrm{Re}[n(\omega)]}} \mathrm{e}^{-\mathrm{Im}[k(\omega)]z} \int_{-\infty}^{z} \mathrm{d}z' \; \mathrm{e}^{-\mathrm{i}k(\omega)z'} \hat{J}_{\mathrm{N}}(z',\omega), \tag{35a}$$

$$\hat{a}_{<}(z,\omega) = \mathrm{i}\sqrt{\frac{\pi\mu_0 cS}{\hbar\omega \mathrm{Re}[n(\omega)]}} \mathrm{e}^{\mathrm{Im}[k(\omega)]z} \int_{z}^{\infty} \mathrm{d}z' \; \mathrm{e}^{\mathrm{i}k(\omega)z'} \hat{J}_{\mathrm{N}}(z',\omega). \tag{35b}$$

They correspond to the annihilation operator of a photon in the dielectric medium, and the commutator is derived as

$$[\hat{a}_{>}(z,\omega), \hat{a}^{\dagger}_{>}(z',\omega)] = [\hat{a}_{<}(z,\omega), \hat{a}^{\dagger}_{<}(z',\omega)] = \delta(\omega-\omega')\mathrm{e}^{-\mathrm{Im}[k(\omega)]|z-z'|} \tag{36}$$

In the loss-less limit ($\mathrm{Im}[\varepsilon(\omega)] \to 0$), they become position-independent and simply considered as the annihilation operator.

Diagonalization of polariton system

The Hamiltonian of polariton system is expressed in Eq. (3). The vector potential and the electric field are expressed by $\hat{a}_k$ as

$$\hat{A}_k = \sqrt{\frac{\hbar}{2\varepsilon_0 c|k|S}}(\hat{a}_k + \hat{a}^{\dagger}_{-k}), \tag{37a}$$

$$\hat{E}_k = \mathrm{i}\sqrt{\frac{\hbar c|k|}{2\varepsilon_0 S}}(\hat{a}_k - \hat{a}^{\dagger}_{k}). \tag{37b}$$

The excitonic polarization and current density are written in k-space as

$$\hat{P}_k = \sqrt{\frac{2\pi\beta\varepsilon_0\hbar\omega_{\mathrm{ex}}}{S}}(\hat{b}_k + \hat{b}^{\dagger}_{-k}), \tag{37c}$$

$$\hat{J}_k = (-\mathrm{i}\omega_{\mathrm{ex}})\sqrt{\frac{2\pi\beta\varepsilon_0\hbar\omega_{\mathrm{ex}}}{S}}(\hat{b}_k - \hat{b}^{\dagger}_{-k}) - 4\pi\beta\varepsilon_0{\omega_{\mathrm{ex}}}^2\hat{A}_k. \tag{37d}$$

The equations of motion are derived as

$$\frac{\partial}{\partial t}\hat{a}_k = -\mathrm{i}c|k|\hat{a}_k + g_k(\hat{b}_k - \hat{b}^{\dagger}_{-k}) - \mathrm{i}2D_k(\hat{a}_k + \hat{a}^{\dagger}_{-k}), \tag{38a}$$

$$\frac{\partial}{\partial t}\hat{b}_k = -\mathrm{i}\omega_{\mathrm{ex}}\hat{b}_k - g_k(\hat{a}_k + \hat{a}^{\dagger}_{-k}). \tag{38b}$$

The equations of motion of the macroscopic fields are obtained as

$$\frac{\partial}{\partial t}\hat{A}_k = -\hat{E}_k, \tag{39a}$$

$$\frac{\partial}{\partial t}\hat{E}_k = c^2k^2\hat{A}_k - \frac{1}{\varepsilon_0}\hat{J}_k, \tag{39b}$$

$$\frac{\partial}{\partial t}\hat{P}_k = \hat{J}_k, \tag{39c}$$

$$\frac{\partial}{\partial t}\hat{J}_k = -{\omega_{\mathrm{ex}}}^2\hat{P}_k - 4\pi\beta\varepsilon_0{\omega_{\mathrm{ex}}}^2\frac{\partial}{\partial t}\hat{A}_k. \tag{39d}$$

The first two equations correspond to the Maxwell equations. Then, the wave equations are obtained as

$$k^2\hat{A}_k + \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\hat{A}_k = \mu_0\hat{J}_k, \tag{40a}$$

$$k^2\hat{E}_k + \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\hat{E}_k = -\mu_0\frac{\partial^2}{\partial t^2}\hat{P}_k. \tag{40b}$$

The equation of motion of the polarization is rewritten as

$$\frac{\partial^2}{\partial t^2}\hat{P}_k = -{\omega_{\mathrm{ex}}}^2\hat{P}_k + 4\pi\beta\varepsilon_0{\omega_{\mathrm{ex}}}^2\hat{E}_k. \tag{41}$$

Then, by Fourier transforming to the frequency domain, we get the dielectric function as

$$\varepsilon(\omega) = \frac{c^2k^2}{\omega^2} = 1 + \frac{4\pi\beta{\omega_{\mathrm{ex}}}^2}{\omega_{\mathrm{ex}} - (\omega+\mathrm{i}0^+)^2} = \frac{{\omega_{\mathrm{exL}}}^2 - \omega^2}{\omega_{\mathrm{ex}} - (\omega+\mathrm{i}0^+)^2}. \tag{42}$$

Here, ω_{exL} is the frequency of longitudinal excitation satisfying

$$4\pi\beta = \frac{{\omega_{\mathrm{exL}}}^2}{{\omega_{\mathrm{ex}}}^2} - 1. \tag{43}$$

The Hamiltonian can be diagonalized by the Bogoliubov transformation. The annihilation operator of lower ($j = L$) or upper ($j = U$) polariton is represented as

$$\hat{p}_{j,k} = w_{jk}\hat{a}_k + x_{jk}\hat{b}_k + y_{jk}\hat{a}^\dagger_{-k} + z_{jk}\hat{b}^\dagger_{-k}. \tag{44}$$

Then, the coefficients are determined by the following eigen-value problem:

$$\begin{pmatrix} c|k| + 2D_k & -\mathrm{i}g_k & -2D_k & -\mathrm{i}g_k \\ \mathrm{i}g_k & \omega_{\mathrm{ex}} & -\mathrm{i}g_k & 0 \\ 2D_k & -\mathrm{i}g_k & -c|k| - 2D_k & -\mathrm{i}g_k \\ -\mathrm{i}g_k & 0 & \mathrm{i}g_k & -\omega_{\mathrm{ex}} \end{pmatrix} \begin{pmatrix} w_{j,k} \\ x_{j,k} \\ y_{j,k} \\ z_{j,k} \end{pmatrix} = \omega_{j,k} \begin{pmatrix} w_{j,k} \\ x_{j,k} \\ y_{j,k} \\ z_{j,k} \end{pmatrix}. \tag{45}$$

The four eigen-states corresponds to $\hat{p}_{j,k}$ and $\hat{p}^\dagger_{j,k}$, whose eigen-frequencies are respectively $\omega_{j,k}$ and $-\omega_{j,k}$:

$$\omega_{L/U,k} = \frac{\omega_{\mathrm{ex}}}{\sqrt{2}}\left\{1 + 4\pi\beta + \frac{c^2|k|^2}{{\omega_{\mathrm{ex}}}^2} \mp \left[\left(1 + 4\pi\beta + \frac{c^2|k|^2}{{\omega_{\mathrm{ex}}}^2}\right)^2 - \frac{4c^2|k|^2}{{\omega_{\mathrm{ex}}}^2}\right]^{1/2}\right\}^{1/2}. \tag{46}$$

The eigen vectors are derived as

$$\begin{pmatrix} w_{L,k} \\ x_{L,k} \\ y_{L,k} \\ z_{L,k} \end{pmatrix} = \left\{\frac{\omega_L}{\omega_{\mathrm{ex}}}\left[\left(1 - \frac{{\omega_L}^2}{{\omega_{\mathrm{ex}}}^2}\right)^2 + 4\pi\beta\right]\right\}^{-1/2} \begin{pmatrix} \left[1 - \frac{{\omega_L}^2}{{\omega_{\mathrm{ex}}}^2}\right]\frac{\omega_L + c|k|}{2\omega_{\mathrm{ex}}}\sqrt{\frac{\omega_{\mathrm{ex}}}{c|k|}} \\ -\mathrm{i}\sqrt{\pi\beta}\left(1 + \frac{\omega_L}{\omega_{\mathrm{ex}}}\right) \\ \left[1 - \frac{{\omega_L}^2}{{\omega_{\mathrm{ex}}}^2}\right]\frac{\omega_L - c|k|}{2\omega_{\mathrm{ex}}}\sqrt{\frac{\omega_{\mathrm{ex}}}{c|k|}} \\ -\mathrm{i}\sqrt{\pi\beta}\left(1 - \frac{\omega_L}{\omega_{\mathrm{ex}}}\right) \end{pmatrix}. \tag{47}$$

$$\begin{pmatrix} w_{U,k} \\ x_{U,k} \\ y_{U,k} \\ z_{U,k} \end{pmatrix} = \left\{\frac{\omega_U}{\omega_{\mathrm{ex}}}\left[\left(1 - \frac{{\omega_U}^2}{{\omega_{\mathrm{ex}}}^2}\right)^2 + 4\pi\beta\right]\right\}^{-1/2} \begin{pmatrix} -\left[1 - \frac{{\omega_U}^2}{{\omega_{\mathrm{ex}}}^2}\right]\frac{\omega_U + c|k|}{2\omega_{\mathrm{ex}}}\sqrt{\frac{\omega_{\mathrm{ex}}}{c|k|}} \\ \mathrm{i}\sqrt{\pi\beta}\left(1 + \frac{\omega_U}{\omega_{\mathrm{ex}}}\right) \\ -\left[1 - \frac{{\omega_U}^2}{{\omega_{\mathrm{ex}}}^2}\right]\frac{\omega_U - c|k|}{2\omega_{\mathrm{ex}}}\sqrt{\frac{\omega_{\mathrm{ex}}}{c|k|}} \\ \mathrm{i}\sqrt{\pi\beta}\left(1 - \frac{\omega_U}{\omega_{\mathrm{ex}}}\right) \end{pmatrix}. \tag{48}$$

We have two eigen-frequencies $\omega_{j,k}$ for a given wavenumber k. Inversely, we get one allowed wavenumber $k(\omega)$ for a given frequency ω. The relation of them are expressed by the dielectric function as

$$\frac{c^2k^2}{{\omega_{j,k}}^2} = \varepsilon(\omega_{j,k}), \tag{49a}$$

$$\frac{c^2k(\omega)^2}{\omega^2} = \varepsilon(\omega). \tag{49b}$$

The Hamiltonian is rewritten as

$$\hat{H} = \sum_{j=L,U} \sum_{k=-\infty}^{\infty} \hbar\omega_{j,k} \hat{p}_{j,k}^{\dagger} \hat{p}_{j,k} \tag{50a}$$

$$= \int_0^{\infty} \mathrm{d}\omega \; \hbar\omega \left[\hat{p}_{>}^{\dagger}(\omega)\hat{p}_{>}(\omega) + \hat{p}_{<}^{\dagger}(\omega)\hat{p}_{<}(\omega) \right], \tag{50b}$$

where the forward and backward polariton operators ($\hat{p}_{>}$ and $\hat{p}_{<}$) are defined in the frequency domain as

$$\hat{p}_{\gtrless}(\omega) = \sqrt{\frac{L}{2\pi v_{\mathrm{g}}(\omega)}} \left[\theta(\omega - \omega_{\mathrm{exL}})\hat{p}_{U,\pm k(\omega)} + \theta(\omega - \omega_{\mathrm{ex}})\hat{p}_{L,\pm k(\omega)} \right]. \tag{51}$$

We choose the phase difference between lower and upper polaritons by the phase of the eigen-vectors shown above for deriving a simple expression below. The group velocity is expressed as

$$v_{\mathrm{g}}(\omega) = \frac{\partial\omega}{\partial k} = \frac{c^2 k}{\omega} \left(\frac{\omega^2}{{\omega_{\mathrm{ex}}}^2} - 1 \right)^2 \left[\left(\frac{\omega^2}{{\omega_{\mathrm{ex}}}^2} - 1 \right)^2 + 4\pi\beta \right]^{-1}. \tag{52}$$

By using the polariton operator, the original photon and excitation operators are expressed as

$$\hat{a}_k = \sum_{j=L,U} (w_{j,k}^{*}\hat{p}_{j,k} - y_{j,k}\hat{p}_{j,-k}^{\dagger}), \tag{53a}$$

$$\hat{b}_k = \sum_{j=L,U} (x_{j,k}^{*}\hat{p}_{j,k} - z_{j,k}\hat{p}_{j,-k}^{\dagger}). \tag{53b}$$

Then, the vector potential at position z is represented as

$$\hat{A}(z) = \frac{1}{\sqrt{L}} \sum_{k=-\infty}^{\infty} \mathrm{e}^{\mathrm{i}kz} \hat{A}_k \tag{54a}$$

$$= \sum_{k=-\infty}^{\infty} \sqrt{\frac{\hbar}{2\varepsilon_0 c|k|SL}} (\hat{a}_k + \hat{a}_{-k}^{\dagger})\mathrm{e}^{\mathrm{i}kz} \tag{54b}$$

$$= \sum_{k=-\infty}^{\infty} \sqrt{\frac{\hbar}{2\varepsilon_0 c|k|SL}} \sum_{j=L,U} \left[(w_{j,k}^{*} - y_{j,-k}^{*})\hat{p}_{j,k} + (w_{j,-k} - y_{j,k})\hat{p}_{j,-k}^{\dagger} \right] \mathrm{e}^{\mathrm{i}kz}. \tag{54c}$$

The positive-frequency and forward component is then written as

$$\hat{A}_{>}^{+}(z) = \sum_{k=0}^{\infty} \sqrt{\frac{\hbar}{2\varepsilon_0 c|k|SL}} \sum_{j=L,U} (w_{j,k}^{*} - y_{j,-k}^{*})\hat{p}_{j,k}\mathrm{e}^{\mathrm{i}kz}. \tag{55}$$

By rewriting the polariton operator in the frequency domain, we get

$$\hat{A}_{>}^{+}(z) = \sum_{j=L,U} \sum_{k=0}^{\infty} \sqrt{\frac{\hbar}{2\varepsilon_0 ckSL}} \frac{\left| 1 - \frac{{\omega_{jk}}^2}{{\omega_{\mathrm{ex}}}^2} \right| \sqrt{\frac{ck}{\omega_{\mathrm{ex}}}}}{\sqrt{\frac{\omega_{jk}}{\omega_{\mathrm{ex}}} \left[\left(1 - \frac{{\omega_{jk}}^2}{{\omega_{\mathrm{ex}}}^2} \right)^2 + 4\pi\beta \right]}} \hat{p}_{j,k}\mathrm{e}^{\mathrm{i}kz} \tag{56a}$$

$$= \sum_{j=L,U} \sum_{k=-\infty}^{\infty} \sqrt{\frac{\hbar v_{\mathrm{g}}(\omega_{jk})}{2\varepsilon_0 c^2 kSL}} \hat{p}_{jk}\mathrm{e}^{\mathrm{i}kz} \tag{56b}$$

$$= \int_0^{\infty} \mathrm{d}\omega \; \sqrt{\frac{\hbar}{4\pi\varepsilon_0 c\omega n(\omega)S}} \; \hat{p}_{>}(\omega)\mathrm{e}^{\mathrm{i}k(\omega)z}. \tag{56c}$$

This has exactly the same form as Eq. (34) in the loss-less limit.

Solution in cavity system by Green's function approach

Let's derive the Green's function $G(z,z',\omega)$ satisfying

$$-\left[\frac{\partial^2}{\partial z^2}+\frac{\omega^2}{c^2}\varepsilon(z,\omega)\right]G(z,z',\omega)=\delta(z-z'). \tag{57}$$

where the dielectric function is expressed as

$$\varepsilon(z,\omega)=\eta(\omega)\delta(z)+\begin{cases}1 & z<0\\ \varepsilon_{\mathrm{p}}(\omega) & 0<z<l\end{cases} \tag{58}$$

Obeying the recipe in Ref. [42, section 2.4], the Green's function $G_{ij}(z,z',\omega)$ (z in region i and z' in region j) is obtained as

$$G_{11}(z,z',\omega)=-\frac{1}{\mathrm{i}2(\omega/c)}\left\{\mathrm{e}^{\mathrm{i}(\omega/c)|z-z'|}+\mathrm{e}^{-\mathrm{i}(\omega/c)z}\tilde{r}_{21}(\omega)\mathrm{e}^{-\mathrm{i}(\omega/c)z'}\right\}, \tag{59a}$$

$$G_{21}(z,z',\omega)=-\frac{1}{\mathrm{i}2(\omega/c)}\sin[k_{\mathrm{p}}(\omega)(l-z)]\tilde{t}_{21}(\omega)\mathrm{e}^{-\mathrm{i}(\omega/c)z'}, \tag{59b}$$

$$G_{12}(z,z',\omega)=-\frac{1}{\mathrm{i}2k_{\mathrm{p}}(\omega)}\mathrm{e}^{-\mathrm{i}(\omega/c)z}\tilde{t}_{12}(\omega)\sin[k_{\mathrm{p}}(\omega)(l-z')], \tag{59c}$$

$$\begin{aligned}G_{22}(z,z',\omega)=&-\frac{1}{\mathrm{i}2k_{\mathrm{p}}(\omega)}\left\{\mathrm{e}^{\mathrm{i}k_{\mathrm{p}}(\omega)|z-z'|}-\mathrm{e}^{-\mathrm{i}k_{\mathrm{p}}(\omega)(z-l)}\mathrm{e}^{-\mathrm{i}k_{\mathrm{p}}(\omega)(z'-l)}\right.\\&\left.+\frac{\mathrm{i}2\mathrm{e}^{\mathrm{i}k_{\mathrm{p}}(\omega)l}[1-\mathrm{i}\Lambda(\omega)-n_{\mathrm{p}}(\omega)]}{[1-\mathrm{i}\Lambda(\omega)]\sin[k_{\mathrm{p}}(\omega)l]+\mathrm{i}n_{\mathrm{p}}(\omega)\cos[k_{\mathrm{p}}(\omega)l]}\sin[k_{\mathrm{p}}(\omega)(l-z)]\sin[k_{\mathrm{p}}(\omega)(l-z')]\right\}\end{aligned} \tag{59d}$$

$$\begin{aligned}=&-\frac{1}{\mathrm{i}2k_{\mathrm{p}}(\omega)}\left\{\mathrm{e}^{\mathrm{i}k_{\mathrm{p}}(\omega)|z-z'|}-\mathrm{e}^{\mathrm{i}k_{\mathrm{p}}(\omega)z}\frac{[1-\mathrm{i}\Lambda(\omega)-n_{\mathrm{p}}(\omega)]\sin[k_{\mathrm{p}}(\omega)(l-z')]}{[1-\mathrm{i}\Lambda(\omega)]\sin[k_{\mathrm{p}}(\omega)l]+\mathrm{i}n_{\mathrm{p}}(\omega)\cos[k_{\mathrm{p}}(\omega)l]}\right.\\&\left.-\mathrm{e}^{-\mathrm{i}k_{\mathrm{p}}(z-l)}\frac{[1-\mathrm{i}\Lambda(\omega)]\sin[k_{\mathrm{p}}(\omega)z']+\mathrm{i}n_{\mathrm{p}}(\omega)\cos[k_{\mathrm{p}}(\omega)z']}{[1-\mathrm{i}\Lambda(\omega)]\sin[k_{\mathrm{p}}(\omega)l]+\mathrm{i}n_{\mathrm{p}}(\omega)\cos[k_{\mathrm{p}}(\omega)l]}\right\},\end{aligned} \tag{59e}$$

$$\tilde{r}_{21}(\omega)=\frac{[1+\mathrm{i}\Lambda(\omega)]\sin[k_{\mathrm{p}}(\omega)l]-\mathrm{i}n_{\mathrm{p}}(\omega)\cos[k_{\mathrm{p}}(\omega)l]}{[1-\mathrm{i}\Lambda(\omega)]\sin[k_{\mathrm{p}}(\omega)l]+\mathrm{i}n_{\mathrm{p}}(\omega)\cos[k_{\mathrm{p}}(\omega)l]}, \tag{60a}$$

$$\tilde{t}_{21}(\omega)=\frac{2}{[1-\mathrm{i}\Lambda(\omega)]\sin[k_{\mathrm{p}}(\omega)l]+\mathrm{i}n_{\mathrm{p}}(\omega)\cos[k_{\mathrm{p}}(\omega)l]}, \tag{60b}$$

$$\tilde{t}_{12}(\omega)=\frac{2n_{\mathrm{p}}(\omega)}{[1-\mathrm{i}\Lambda(\omega)]\sin[k_{\mathrm{p}}(\omega)l]+\mathrm{i}n_{\mathrm{p}}(\omega)\cos[k_{\mathrm{p}}(\omega)l]}. \tag{60c}$$

The derivation is as follows. When a source exists in region 1, the Green's function can be supposed as follows:

$$G_{j1}(z,z',\omega)=-\frac{\mathcal{G}_{j1}(z,z',\omega)}{\mathrm{i}2(\omega/c)}, \tag{61}$$

$$\mathcal{G}_{11}(z,z',\omega)=\mathrm{e}^{\mathrm{i}(\omega/c)|z-z'|}+\mathrm{e}^{-\mathrm{i}(\omega/c)z}B_{11}(z'), \tag{62a}$$

$$\mathcal{G}_{21}(z,z',\omega)=\mathrm{e}^{\mathrm{i}k_{\mathrm{p}}(\omega)(z-l)}F_{21}(z')+\mathrm{e}^{-\mathrm{i}k_{\mathrm{p}}(\omega)(z-l)}B_{21}(z'). \tag{62b}$$

We get a boundary condition at $z=l$:

$$F_{21}(z')+B_{21}(z')=0. \tag{63a}$$

At $z=0$, we also get

$$\mathrm{e}^{-\mathrm{i}(\omega/c)z'}+B_{11}(z')=\mathrm{e}^{-\mathrm{i}k_{\mathrm{p}}(\omega)l}F_{21}(z')+\mathrm{e}^{\mathrm{i}k_{\mathrm{p}}(\omega)l}B_{21}(z'), \tag{63b}$$

$$\mathrm{e}^{-\mathrm{i}(\omega/c)z'}-B_{11}(z')-n_{\mathrm{p}}(\omega)\left[\mathrm{e}^{-\mathrm{i}k_{\mathrm{p}}(\omega)l}F_{21}(z')-\mathrm{e}^{\mathrm{i}k_{\mathrm{p}}(\omega)l}B_{21}(z')\right]=-\mathrm{i}\Lambda(\omega)\left[\mathrm{e}^{-\mathrm{i}(\omega/c)z'}+B_{11}(z')\right]. \tag{63c}$$

The third condition is obtained by the boundary condition (17b) or integrating Eq. (61). Solving them, we get G_{11} and G_{21}.

When a source exists at region 2, we can suppose

$$G_{j2}(z, z', \omega) = -\frac{\mathcal{G}_{j2}(z, z', \omega)}{\mathrm{i}2k_{\mathrm{p}}(\omega)}, \tag{64}$$

$$\mathcal{G}_{12}(z, z', \omega) = \mathrm{e}^{-\mathrm{i}(\omega/c)z} B_{12}(z'), \tag{65a}$$

$$\mathcal{G}_{22}(z, z', \omega) = \mathrm{e}^{\mathrm{i}k_{\mathrm{p}}(\omega)|z-z'|} + \mathrm{e}^{\mathrm{i}k_{\mathrm{p}}(\omega)z} F_{22}(z') + \mathrm{e}^{-\mathrm{i}k_{\mathrm{p}}(\omega)(z-l)} B_{22}(z'). \tag{65b}$$

At $z = l$, we get

$$\mathrm{e}^{\mathrm{i}k_{\mathrm{p}}(\omega)(l-z')} + \mathrm{e}^{\mathrm{i}k_{\mathrm{p}}(\omega)l} F_{22}(z') + B_{22}(z') = 0. \tag{66a}$$

Further, at $z = 0$

$$\mathrm{e}^{\mathrm{i}k_{\mathrm{p}}(\omega)z'} + F_{22}(z') + \mathrm{e}^{\mathrm{i}k_{\mathrm{p}}(\omega)l} B_{22}(z') = B_{12}(z'), \tag{66b}$$

$$B_{12}(z') + n_{\mathrm{p}}(\omega)\left[-\mathrm{e}^{\mathrm{i}k_{\mathrm{p}}(\omega)z'} + F_{22}(z') - \mathrm{e}^{\mathrm{i}k_{\mathrm{p}}l} B_{22}(z')\right] = \mathrm{i}\Lambda(\omega) B_{12}(z'). \tag{66c}$$

Then, G_{22} and G_{12} are obtained.

By using the Green's function, the vector potential is obtained in region 1 as

$$\begin{aligned}\hat{A}_1^+(z, \omega) = {} & \mathrm{e}^{\mathrm{i}(\omega/c)z} \hat{A}_{1>}^+(z, \omega) + \mathrm{e}^{-\mathrm{i}(\omega/c)z} \tilde{r}_{21} \hat{A}_{1>}^+(0, \omega) \\ & - \frac{\mu_0}{\mathrm{i}2(\omega/c)} \int_z^0 \mathrm{d}z'\ \mathrm{e}^{-\mathrm{i}(\omega/c)(z-z')} \hat{J}_{\mathrm{N}}(z') + \mu_0 \int_0^l \mathrm{d}z'\ G_{12}(z, z', \omega) \hat{J}_{\mathrm{N}}(z'). \end{aligned} \tag{67}$$

where we define the incoming field as

$$\hat{A}_{1>}^+(z, \omega) = -\frac{\mu_0}{\mathrm{i}2(\omega/c)} \int_{-\infty}^z \mathrm{d}z'\ \mathrm{e}^{-\mathrm{i}(\omega/c)z'} \hat{J}_{\mathrm{N}}(z'). \tag{68}$$

Here, as discussed in Ref. [22], we can focus only on the incoming and outgoing fields at the boundary $z = 0$, because the contribution such as the third term in Eq. (67) can be neglected in the loss-less limit (if there is no dissipation, we need not consider the noise operator for the propagation in vacuum). Further, the last term can also be neglected in the loss-less limit, then the vector potential in region 1 can be expressed as

$$\hat{A}_1^+(z, \omega) = \mathrm{e}^{\mathrm{i}(\omega/c)z} \hat{A}_{1>}^+(\omega) + \mathrm{e}^{-\mathrm{i}(\omega/c)z} \hat{A}_{1<}^+(\omega). \tag{69}$$

The first term corresponds to the incoming field $\hat{A}_{1>}^+(\omega) = \hat{A}_{1>}^+(0, \omega)$, and the second term is the outgoing field represented as

$$\hat{A}_{1<}^+(\omega) = \tilde{r}_{21} \hat{A}_{1>}^+(0, \omega). \tag{70}$$

On the other hand, the vector potential in region 2 is obtained in the loss-less limit as

$$\hat{A}_2^+(z, \omega) = \tilde{t}_{21} \hat{A}_{1>}^+(\omega) \sin[k_{\mathrm{p}}(\omega)(l - z)]. \tag{71}$$

Then, we get

$$\hat{A}_2^+(\omega) = \tilde{t}_{21} \hat{A}_{1>}^+(\omega). \tag{72}$$

These fields $\hat{A}_{1>}^+(\omega)$, $\hat{A}_{1<}^+(\omega)$, $\hat{A}_2^+(\omega)$ certainly satisfy the Maxwell's boundary conditions.

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[29] Even if we suppose the expression such as $(\hat{a}\pm\hat{a}^\dagger)(\hat{\alpha}\mp\hat{\alpha}^\dagger)$ instead of Eq. (1) [15], $\kappa_{L,U}^{\mathrm{RWA}}$ shows qualitatively different behaviors compared to Eq. (21).

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